Dynamic Three-Factor Models of International Trade*

Yunfang Hu[a#], Kazuo Nishimura[b†], and Koji Shimomura[c♦]

[a] *Graduate School of Economics, Kobe University*

[b] *Institute of Economic Research, Kyoto University*

[c] *Research Institute for Economics and Business Administration, Kobe University*

Abstract

Based on the Jones (1971) model, we construct two dynamic models of international trade in which the rate of time preference is either constant or time-varying. The main purpose is to study whether and under what conditions the results derived from the Jones model still hold in the dynamic framework. It is shown that the results of dynamic models may be similar or different to those obtained in the static model. For example, it is possible that, in both static and dynamic models, an increase in a commodity price raises this commodity's output and the rate of return to the specific factor in this sector. However, the effect on the wage rate may be different due to the factor accumulation impact in the dynamic framework.

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1. Introduction

In the seminal 1971 paper, Ronald W. Jones developed a specific factor model which is regarded as one of the basic general equilibrium models of international trade. Subsequently, this three-factor model has been developed in many ways. For example, it has been used intensively in discussing the short-run issues of international trade.[1] Though this literature is large, there are few studies, to our knowledge, examining the dynamic

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* This paper is most respectfully dedicated to Professor Ronald Jones.

hu@econ.kobe-u.ac.jp

† nishimura@kier.kyoto-u.ac.jp.

♦ simomura@rieb.kobe-u.ac.jp.

[1] Among others, see Mayer (1974), Mussa (1974), and Neary (1978), for example.

property of the above general equilibrium model. In order to extend the Jones model to the dynamic framework, in the present paper we assume that the general production factor is accumulable and households allocate their income intertemporally. The main purpose is to study whether and under what conditions the results derived from the Jones model still hold in the dynamic frameworks.

The reason we investigate the specific factor model in a dynamic framework is that there exist some production factors that cannot move across sectors even in the long run. This can happen either because of a large transportation cost, or because of differences in the factors they use in production processes. For example, if specifying the two industries as agriculture and manufacturing, and their productions need labor-land and labor-capital as production factors, respectively, then in this case labor is the mobile factor while capital and land are not.

The Jones model can be extended in different ways. In order to obtain a dynamic counterpart to the Jones model, we specify the mobile factor as human capital, which can be fostered by education. The human capital literature tells us that there are two components in education-related expenditure, investment and consumption (Shultz, 1963). That is, households' education expenditure can, on the one hand, raise the level of human capital, and, on the other hand, it can augment households' utility through consumption of the education service directly.

Two dynamic models will be presented here. In the first one, we extend the Jones model to a dynamic framework by assuming an accumulable general factor and a constant rate of time preference. This model is refined in the second model by assuming that the rate of time preference depends on the utility level. This kind of variable rate of time preference was originally considered by Uzawa (1968) and its relevance in improving economic models has been proved by empirical studies.[2] In both models, we prove the existence, uniqueness and saddle-point stability of the steady state. Next, we conduct comparative statics with respect to the commodity prices. Then we compare the results with those obtained in the static framework.

The rest of the paper is arranged as follows. Section 2 reconstructs the Jones model. Section 3 presents a dynamic version of the Jones model in which the rate of the time preference is constant. This model is modified in Section 4 by endogenizing the rate of time preference. The proofs of the paper's main results can be found in the Appendix.

2. The Jones Model

In this section, we reformulate the Jones model with duality. Following the convention, we consider a world economy consisting of a large number of countries. Each country is small in the sense that it takes the world price as given. Produced commodities are internationally tradable, while production factors are not.

There are two production sectors in which two internationally tradable goods, Goods 1 and 2, are produced. Good 1 is a pure consumption good with price p, while Good 2 is a

[2] A recent paper by Guest and McDonald (2001) shows how the Uzawa preference improves the simulations properties of the small open economy world.

consumable capital good which serves as the numeraire. Good i (i = 1, 2) is produced by using a sector-specific factor of production L_i and a general factor, K. While L_i is given and constant over time, K is the accumulated Good 2. As we mentioned in the introduction, the general factor K is human capital and sector 2 is the education sector. Households' education expenditure can, on the one hand, increase productivity through human capital accumulation, and, on the other hand, provide utility to households by consuming those services directly.

Following the Jones model, we assume neoclassical production technologies in both sectors. Let us denote the production function of Good i by $Y_i = F^i(K_i, L_i)$. Denote the wage to the general factor K, and the returns to the sector-i specific factors L_i as w, r_i respectively. Then market competition makes each factor's marginal production equal to its rate of return. That is, $w = pF_1^1 = F_1^2$, $r_1 = pF_2^1$ and $r_2 = F_2^2$.

For given aggregate amounts of general factor K and specific forces L_1, L_2, we have the GDP function

$$G(p, K, L_1, L_2) \equiv \max_{K_i} \{pF^1(K_1, L_1) + F^2(K_2, L_2) \,|\, K_1 + K_2 \leq K\}$$
$$= pF^1(K_1(p, K, L_1, L_2), L_1) + F^2(K_2(p, K, L_1, L_2), L_2)$$

where $(K_1(p, K, L_1, L_2), K_2(p, K, L_1, L_2))$ is the optimal solution. From the constant-return-to-scale property of the production functions, the GDP function has the following properties.[3]

$$G_p = F^1,\ G_K = pF_1^1 = F_1^2 = w,\ G_{L_1} = pF_2^1 = r_1,\ G_{L_2} = F_2^2 = r_2$$

$$G_{pp} > 0,\ G_{Kp} = G_{pK} > 0,\ G_{KK} < 0$$

$$G_{L_1p} > 0,\ G_{L_2p} < 0,\ G_{L_1K} > 0,\ G_{L_2K} > 0$$

That is, GDP function's partial derivative with respect to the commodity price p is equal to the output of this good, its partial derivatives with respect to the factor endowments are the same as the value of these factors' marginal productivity in each sector.

Define the representative household's momentary utility function as $u = U(c_1, c_2)$, where c_i is the consumption of Good i. We assume that the utility function satisfies the standard properties and also homotheticity. That is, $U(c_1, c_2)$ can be rewritten as the composite function of a strictly increasing function $g(\upsilon)$ and a linearly homogeneous function $\upsilon(c_1, c_2)$: $U(c_1, c_2) \equiv g(\upsilon(c_1, c_2))$.

Lemma 1 *$g(\upsilon)$ is strictly concave if and only if $U(c_1, c_2)$ is strictly concave.*
Proof. *See Appendix 2.*

[3] See Appendix 1 for the proof.

For given preference level u, the representative household's expenditure function is

$$E(p, u) \equiv \min_{c_i} \{pc_1 + c_2 \,|\, u \leq U(c_1, c_2) \equiv g(\upsilon(c_1, c_2))\}.$$

From the homothetic property of the utility function, the following results can be derived.

Lemma 2 *If the momentary utility function $U(c_1, c_2)$ is homothetic, then the expenditure function is multiplicatively separable,*

$$E(p, u) \equiv e(p)\phi(u)$$

where $e(p)$ is increasing and concave in p, while $\phi(u)$, the inverse function of $g(\upsilon)$, is increasing and convex in u. That is

$$e' > 0, e'' < 0, \phi' > 0, \phi'' > 0.$$

Proof. *See Appendix 3.*

In the original Jones model, factor endowments, K and L_i $(i = 1, 2)$ are fixed. Since no intertemporal allocation exists, goods produced at each point of time t are completely consumed at that time. At equilibrium, the model can be summarized in the following condition:

$$e(p)\phi(u) = G(p, K, L_1, L_2) \tag{1}$$

which determines the maximum level u for given p, K, and L_i $(i = 1, 2)$.

For given K, L_i $(i = 1, 2)$, the effects of a change in commodity price p upon outputs and factor prices are shown as

$$\partial Y_1 / \partial p = G_{pp} > 0 \tag{2a}$$

$$\partial w / \partial p = G_{Kp} > 0 \tag{2b}$$

$$\partial r_1 / \partial p = G_{L1p} > 0 \tag{2c}$$

The relations in (2) make clear the effects of price change upon outputs and factor prices. It is shown that a price increase of Good 1 can raise all the factor prices in that sector. Notice also that, preference parameters do not affect those comparative static results that rely solely on the production side of the economy.

As in Jones (1971), we can derive the following factor-price/commodity-price relation by using the properties of the GDP function (See Appendix 4 for the proof).

$$dr_1/r_1 > dp/p > dw/w > 0 > dr_2/r_2 \tag{3}$$

That is, there is no more magnification effect for the mobile factor.

3. The dynamic model with constant rate of time preference

When the mobile factor is accumulable, households are able to maximize their aggregate utility intertemporally. The representative household's optimization problem is

$$\max_{u} \int_{0}^{+\infty} u e^{-\rho t} dt$$

subject to

$$\dot{K} = G(p, K, L_1, L_2) - e(p)\phi(u)$$

for $K(0)$ given.

The Hamiltonian is $H \equiv u + \lambda[G(p, K, L_1, L_2) - e(p)\phi(u)]$ and at the interior solutions, the necessary conditions for optimality are

$$\partial H/\partial u = 1 - \lambda e(p)\phi'(u) = 0,$$

$$\dot{\lambda} = \lambda[\rho - G_K].$$

Then the economy as a whole can be fully described by the following dynamic system

$$\dot{K} = G(p, K, L_1, L_2) - e(p)\phi(u) \tag{4}$$

$$\dot{\lambda} = \lambda[\rho - G_K(p, K, L_1, L_2)] \tag{5}$$

$$0 = 1 - \lambda e(p)\phi'(u) \tag{6}$$

for which the steady-state conditions are

$$G(p, K, L_1, L_2) = e(p)\phi(u) \tag{7}$$

$$G_K(p, K, L_1, L_2) = \rho \tag{8}$$

Then we have the following result.

Proposition 1 *There is one and only one steady state in this economy which is locally saddle-point stable.*

Proof. *See Appendix 5.*

Furthermore, the relation in (8) implies that a constant wage rate must be realized at the steady state.

Comparative statics

Let us consider how an exogenous change in the commodity price affects the outputs and factor prices. Since K is an endogenous variable, we have, from (8),

$$G_{KK}dK = -G_{Kp}dp \tag{9}$$

By the properties of the GDP function, a price increase in the consumption goods lowers the level of the general factor in the long run.

Using the above result, we obtain the following comparative static results with respect to production.

$$\partial Y_1/\partial p = G_{pp} - (G_{Kp})^2/G_{KK} > 0, \tag{10a}$$

$$\partial w/\partial p = 0 \tag{10b}$$

$$\partial r_1/\partial p = G_{L_1p} - G_{L_1K}G_{Kp}/G_{KK} > 0 \tag{10c}$$

We find that: (i) as in the static model, an increase in commodity price p raises the output and the rate of return to the specific factor in this sector, (ii) unlike the static model, the steady state wage rate is restrained to the constant rate of time preference. This result will be modified in the second dynamic model.

4. The dynamic model with variable rate of time preference

We have seen in the preceding section that, when the rate of time preference ρ is constant, the rate of return to the general factor is fixed. Hence, any change in the commodity price will not affect the wage rate. Furthermore, households' preference parameters do not affect the output and factor prices in the case of a commodity price change. We will show in the following that these results can be modified by introducing a variable rate of time preference.[4]

Following Uzawa (1968), we assume the rate of time preference ρ is a positive, increasing, and convex function of the instantaneous utility level u. That is, for all $u > 0$,

$$\rho(u) > 0,\ \rho'(u) > 0,\ \rho''(u) > 0,\ \rho(u) - u\rho'(u) > 0$$

and $\rho(0) > 0$. If denote $X(t) = e^{-\rho(u)t}$, then we have the discount term X satisfying $\dot{X} = -\rho(u)X$ and $X(0) = 1$. Therefore, the representative household's optimization problem is for $K(0)$ given,

$$\max_u \int_0^{+\infty} uX(t)dt$$

subject to

$$\dot{K} = G(p, K, L_1, L_2) = e(p)\phi(u)$$

$$\dot{X} = -\rho(u)X,\ X(0) = 1.$$

[4] A recent paper by Chen, Nishimura and Shimomura (2004) used a variable rate of time preference in order to pin down a unique steady state.

The current value Hamiltonian of the above optimization problem is $H \equiv uX + \lambda[G(p, K, L_1, L_2) - e(p)\phi(u)] - \theta\rho(u)X$, where λ and θ are the co-state variables related to the human capital K and the discount term X. At interior solutions, the necessary conditions for optimization are

$$\dot{K} = G(p, K, L_1, L_2) = e(p)\phi(u)$$

$$\dot{X} = -\rho(u)X$$

$$\dot{\lambda} = -\lambda G_K(p, K, L_1, L_2)$$

$$\dot{\theta} = -u + \theta\rho(u)$$

$$0 = X - \lambda e(p)\phi'(u) - \theta\rho'(u)X$$

Notice that the right-hand side of the last equation is linearly homogeneous in λ and X.

Defining a new variable $\xi \equiv \lambda/X$, we can rewrite the above system as

$$\dot{K} = G(p, K, L_1, L_2) = e(p)\phi(u) \tag{11}$$

$$\dot{\xi} = \xi[\rho(u) - G_K(p, K, L_1, L_2)] \tag{12}$$

$$\dot{\theta} = -u + \theta\rho(u) \tag{13}$$

$$0 = 1 - \xi E(p)\phi'(u) - \theta\rho'(u) \tag{14}$$

The steady-state conditions for K and u are

$$G(p, K, L_1, L_2) = e(p)\phi(u) \tag{15}$$

$$G_K(p, K, L_1, L_2) = \rho(u) \tag{16}$$

and the steady state value of θ is determined by $u = \theta\rho(u)$.

Proposition 2 *There is one and only one steady state satisfying (15) and (16) which is locally saddle-point stable.*

Proof. *See Appendix 6.*

Comparative statics

For given specific factor endowments, totally differentiate the two sides of (15) and (16) with respect to K, p and u to yield

$$\left[e\phi' - \frac{G_K}{G_{KK}}\rho'\right] dK = \left\{\left[G_p - e'(p)\phi(u)\right]\frac{\rho'}{G_{KK}} - \frac{G_{Kp}}{G_{KK}}e\phi'\right\}dp \tag{17}$$

Notice that, the varying effect of the rate of time preference ρ' enters the two sides of (17), which makes the trade pattern affect on the general factor K when commodity price p changes. It is clear that an increase in p raises K when the country imports the consumption goods. Otherwise, if the country exports the consumption goods, generally there is no clear-cut result since the two terms on the right-hand side of the above expression have opposite signs.

Proposition 3 *When the country imports the pure consumption good (i.e., $G_p - e'(p)\phi(u) < 0$), or exports this good in a small amount (i.e., $G_p - e'(p)\phi(u)$ is close to 0), then a price increase in this good will increase the accumulation of the general factor K.*

Based on the above result, we can derive the effects of commodity price change upon outputs and factor prices.

$$\partial Y_1/\partial p = G_{pp} + G_{Kp}dk / dp > 0 \tag{18a}$$

$$\partial w/\partial p = G_{Kp} + G_{KK}dK / dp \tag{18b}$$

$$\partial r_1/\partial p = G_{L_1p} + G_{L_1K}dK / dp > 0 \tag{18c}$$

Notice that, similar to the preceding models, an increase in commodity price p raises (conditionally) the output and the rate of return to the specific factor of this sector, Y_1 and r_1. As for the effect of this price change on the wage rate, the result is generally ambiguous. On the one hand, an increase in p raises the wage rate directly, as in the Jones model, on the other hand, however, this increase in p has an indirect decreasing effect on the wage rate through its impact on K. Depending on the relative scales of these two effects, an increase in p can raise or lower the wage rate in the long run.

Conclusion

In this paper, we extended the Jones (1971) model to two dynamic frameworks with constant and variable rates of time preference. We found that, when households face a constant rate of time preference, comparative static results similar to those in the static model can be obtained.

When households' rate of time preference is time-varying, as long as the country imports the consumption good or exports a small amount of this good, we can still obtain results similar to those in the Jones model.

Appendix

1. *Properties of the GDP function $G(p, K, L_1, L_2)$*

Let $\mathcal{L} \equiv pF^1(K_1, L_1) + F^2(K_2, L_2) + \lambda(K - K_1 - K_2)$, then at the interior points $pF_1^1 = F_1^2$ $(=\lambda)$ and $K_1 + K_2 = K$ from which we obtain the optimal solution $K_1 = K_1(p, K, L_1, L_2)$, $K_2 = K_2(p, K, L_1, L_2)$. Notice that at the optimal point

$$pF_1^1(K_1(p, K, L_1, L_2), L_1) = F_2^1(K - K_1(p, K, L_1, L_2), L_2)$$

always holds. Hence

$$\partial K_1/\partial p = -F_1^1(pF_{11}^1 + F_{11}^2)^{-1},\ \partial K_1/\partial K = F_{11}^2(pF_{11}^1 + F_{11}^2)^{-1},$$
$$\partial K_1/\partial L_1 = -F_{12}^1(pF_{11}^1 + F_{11}^2)^{-1},\ \partial K_1/\partial L_2 = F_{12}^2(pF_{11}^1 + F_{11}^2)^{-1} \tag{19}$$

Since $G(p, K, L_1, L_2) = pF^1(K_1(p, K, L_1, L_2), L_1) + F^2(K_2(p, K, L_1, L_2), L_2)$, then

$$G_p = F^1(p, K, L_1, L_2) + \lambda(\partial K_1/\partial p + \partial K_2/\partial p)$$

Notice that $K_1(p, K, L_1, L_2) + K_2(p, K, L_1, L_2) = K$ and K is given, thus $G_p = Y_1$.

Similarly,

$$G_K = F_1^1 = F_1^2 = w,\ G_{L_1} = pF_2^1 = r_1,\ G_{L_2} = F_2^2 = r_2$$

Combining these relations with (19), we obtain

$$G_{pp} = -(F_1^1)^2(pF_{11}^1 + F_{11}^2)^{-1},\ G_{KK} = pF_{11}^1 F_{11}^2(pF_{11}^1 + F_{11}^2)^{-1}$$

$$G_{Kp} = G_{pK} = F_1^1 F_{11}^2(pF_{11}^1 + F_{11}^2)^{-1},$$

$$G_{L_1p} = F_2^1 - pF_{21}^1 F_1^1(pF_{11}^1 + F_{11}^2)^{-1},\ G_{L_2p} = F_{21}^2 F_1^1(pF_{11}^1 + F_{11}^2)^{-1}$$

$$G_{L_1K} = pF_{21}^2 F_{11}^2(pF_{11}^1 + F_{11}^2)^{-1},\ G_{L_2K} = F_{21}^2 F_{11}^2(pF_{11}^1 + F_{11}^2)^{-1}$$

2. *Proof of Lemma 1*

If $U(c_1, c_2)$ is a quasi-concave function and satisfies $U_1 > 0$, $U_2 > 0$, then $U_{11}U_2^2 + U_1^2U_{22} - 2U_1U_2U_{12} < 0$. Using this result and the monotonicity of $g(\upsilon)$, it is straightforward that $\upsilon(c_1, c_2)$ satisfies

$$\upsilon_{11}\upsilon_2^2 + \upsilon_1^2\upsilon_{22} - 2\upsilon_1\upsilon_2\upsilon_{12} = (g')^{-3}(U_{11}U_2^2 + U_1^2U_{22} - 2U_1U_2U_{12}) < 0$$

that is, $\upsilon(c_1, c_2)$ is a quasi-concave function too. Since, in addition, it is linearly homogenous, then $\upsilon(c_1, c_2)$ satisfies $\upsilon_{11} \leq 0$, $\upsilon_{22} \leq 0$ *and* $\upsilon_{11}\upsilon_{22} - (\upsilon_{12})^2 = 0$.

Since

$$U_{11} = g''(\upsilon)\upsilon_1^2 + g'(\upsilon)\upsilon_{11},\ U_{22} = g''(\upsilon)\upsilon_2^2 + g'(\upsilon)\upsilon_{22}$$

$$U_{12} = U_{21} = g''(\upsilon)\upsilon_1\upsilon_2 + g'(\upsilon)\upsilon_{12}$$

$$U_{11}U_{22} - U_{12}U_{21} = g'(\upsilon)g''(\upsilon)(\upsilon_{11}\upsilon_2^2 + \upsilon_1^2\upsilon_{22} - 2\upsilon_1\upsilon_2\upsilon_{12} + [g'(\upsilon)]^2 [\upsilon_{11}\upsilon_{22} - (\upsilon_{12})^2]$$

then $g''(\cdot) < 0$ if $U(c_1, c_2)$ satisfies $U_{11}U_{22} - U_{12}U_{21} > 0$, and $U_{11}U_{22} - U_{12}U_{21} > 0$, $U_{11} < 0$, $U_{22} < 0$ *if* $g''(\cdot) < 0$. That is, $g(\upsilon)$ is strictly concave if and only if $U(c_1, c_2)$ is strictly concave.

3. *Proof of Lemma 2*

From the definition of the expenditure function

$$E(p, u) \equiv \min_{c_i} \{pc_1 + c_2 \,|\, u \leq U(c_1, c_2) \equiv g(\upsilon(c_1, c_2))\}\ .$$

we have the optimal c_i which satisfies

$$p = \upsilon_1(c_1, c_2)/\upsilon_2(c_1, c_2).$$

Remember that $\upsilon(c_1, c_2)$ is linearly homogenous, then the above relation can be rewritten as $c_1/c_2 = \psi(p)$. Substitute this relation into $u = U(c_1, c_2)$, that is, $g^{-1}(u) = \upsilon(c_1, c_2)$ to obtain $g^{-1}(u) = c_2\upsilon(\psi(p), 1)$, or $c_2 = g^{-1}(u)/\upsilon(\psi(p), 1)$. Notice that, at the optimal point,

$$\frac{\partial c_1}{\partial p} = \frac{g'\upsilon_2}{\upsilon_2^2\upsilon_{11} - 2\upsilon_1\upsilon_2\upsilon_{12} + \upsilon_1^2\upsilon_{22}} < 0$$

For the optimal $c_1(p, u)$, $c_2(p, u)$

$$E(p, u) = pc_1(p, u) + c_2(p, u) \equiv e(p)\phi(u)$$

where $e(p) = [p\psi(p) + 1]/\upsilon(\psi(p), 1)$ and $\phi(u) = g^{-1}(u)$. Notice that, $E_p(p, u) = c_1(p, u)$, and $\partial c_1/\partial p < 0$, we have $e(p)$ is increasing and concave. While $\phi(u)$, the inverse function of $g(\upsilon)$, is increasing and convex in u. That is

$$e' > 0,\ e'' < 0,\ \phi' > 0,\ \phi'' > 0.$$

4. *Proof of the factor-price/commodity-price relationship in (3)*

From

$$G_{L_1} - pG_{L_1p} = pF_2^1 - p\,[F_2^1 - pF_{21}^1 F_1^1 (pF_{11}^1 + F_{11}^2)^{-1}] > 0$$

$$G_K - pG_{pK} = pF_1^2 F_{11}^1 (pF_{11}^1 + F_{11}^2)^{-1} > 0$$

we have $dr_1/r_1 > dp/p > dw/w$ if $dp/p > 0$.While $G_{Kp} > 0$ and $G_{L_2p} < 0$ mean $\partial w/\partial p > 0$ and $\partial r_2/\partial p < 0$ respectively, which mean in turn that $dw/w > 0$ and $dr_2/r_2 < 0$ if $dp/p > 0$, then we have the relation in (3).

5. *Steady state analysis of the dynamic model with constant rate of time preference*

At the steady state, (7) and (8) hold. As G_K is monotonic in K, given p, L_1, L_2 and ρ, (8) determines a unique steady state of K. Substitute K's steady state value into (7), a unique steady state value of u can be derived.

To examine the stability of the above steady state, from the dynamic system, we have

$$\dot{K}_p(t, p) = G_K(K, p)K_p(t, p) - e(p)\phi'(u)u_p(t, p) + [G_p(K, p) - e_p(p)\phi(u)]$$

$$\dot{\lambda}_p(t, p) = -\lambda G_{KK}(K, p)K_p(t, p) - \lambda G_{Kp}(K, p)$$

$$0 = -e(p)\phi'(u)\,\lambda_p(t, p) - \lambda e(p)\phi''(u)u_p(t, p) - \lambda e_p(p)\phi'(u)$$

Therefore the eigen equation of the dynamic system is

$$\Psi(x) = \begin{vmatrix} G_K - x & 0 & -e\phi' \\ -\lambda G_{KK} & -x & 0 \\ 0 & -e\phi' & -\lambda e\phi'' \end{vmatrix}$$

$$= -\lambda e\phi'' x^2 + G_K \lambda e\phi'' x - \lambda G_{KK}(e\phi')^2$$

which has a positive and a negative root, respectively. Since the dynamic system of the model is constituted by a state variable K and a control variable λ, then the unique steady state in this economy is locally saddle-point stable.

6. *Steady state analysis of the dynamic model with variable rate of time preference*

For a given price and labor endowments, (15) reveals a positive relation between the steady state values of K and u, i.e., $\partial u/\partial K > 0$. Since K is indispensable in production and

ϕ is increasing in u, then a smaller human-capital stock level K should correspond to a lower utility level u. On the other hand, from (16), $\partial u/\partial K > 0$. Since a smaller K corresponds to a higher level of G_K, and ρ is increasing in u, therefore a sufficiently small K will now lead to a large u. The above two relations together determine a unique steady state.

Linearizing the dynamic system (11)-(14) around the above steady state, the following characteristic equation of the linear system is obtained.

$$\Gamma(x) \equiv \begin{vmatrix} G_K - x & 0 & 0 & -e\phi' \\ -\xi G_{KK} & -x & 0 & \xi\rho' \\ 0 & 0 & \rho - x & -1 + \theta\rho' \\ 0 & -e\phi' & -\rho' & \Delta \end{vmatrix} = 0$$

where $\Delta \equiv -\xi e\phi'' - \theta\rho'' < 0$. Since $-\xi e\phi' = -1 + \theta\rho'$ and at the steady state, $G_K = \rho$, we have

$$\Gamma(x) = (\rho - x)\,[\Delta x^2 - \rho\Delta x + e\phi'\xi\,(\rho'\rho - G_{KK}e\phi')]$$

Note that $\Delta < 0$ and $e\phi'\xi\,(\rho'\rho - G_{KK}e\phi') > 0$. Hence, the linearized dynamic system has one stable and two unstable eigenvalues, which mean the steady state is saddle-point stable.

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